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The Micro-Foundations of Intertemporal Price Discrimination

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Abstract

This paper investigates the optimality of intertemporal price discrimination for a durable-good monopoly in a model where infinitely-lived households face an intertemporal budget constraint, and consume both durable goods and non-durable goods. We prove that the optimal price of the durable good is not constant, and may decrease or increase over time. Some households may choose to purchase the durable good at a later date, and pay lower or higher prices, since the gain in discounted utility of consuming more of the non-durable good more than compensates for the loss in utility from delaying the consumption of the durable good.

Key Words and Phrases: Intertemporal price discrimination, durable good monopoly, optimal pricing strategy, household demand

JEL Classification: D40, D42, D91

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1. Introduction

To maximize profits, firms utilize whatever market power they possess to pursue differential pricing policies. The success of these sales strategies to extract consumer surplus depends on the extent of heterogeneity among consumers – e.g. household composition, budget constraints and utilities derived from the product – as well as the effectiveness of measures to prevent resale arbitrage. In the case of *static* price discrimination, it is well-known that price discrimination is both feasible and profitable for a monopoly, as long as arbitrage and resale is deterred. In the case of intertemporal price discrimination (henceforth, IPD for short), the typical strategy is to charge higher prices for a product initially, and lower prices in future periods, even though there may be no cost improvements to prompt the price reductions. Such a pricing strategy allows firms with monopoly power to generate higher profits compared with a commitment to charge the same price across all time periods.[1]

The theoretical modeling of IPD for a durable-good monopoly has to contend with the fact that if consumers rationally expect prices to fall over time, this would induce some consumers to optimally delay their purchase. By doing so, this reduces the profitability for the monopoly, and places a constraint on the price that the monopoly can charge at each point in time. This issue was first raised in Coase (1972); as the period over which a durable-good monopoly is able to make binding sales commitment diminishes, its ability to exert market power decreases correspondingly. In the limit, as the feasible period of commitment becomes infinitesimal, the monopoly's market power vanishes, and it is unable to price-discriminate. The competitive market outcome occurs even if the durable good is supplied by the monopoly.[2]

Since then, the literature has developed in two broad directions.[3] One line of research studies the situations under which the Coase conjecture applies or do not apply.[4] A second line of research examines the circumstances under which IPD is optimal for a durable-good monopoly that is able to commit to its pricing policies. This line of research includes Stokey

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[1] Examples abound of IPD in marketing strategies; for instance, the sequential release of hard-cover and soft-cover book editions, and movie screenings in cinemas, then released on video. In practice, intertemporal price discrimination is linked to many factors as well. For instance, creating different versions of a product sequentially and selling limited quantities of a product over time are two strategies frequently used to price-discriminate over time. These strategies are often intertwined with quality improvements and cost reduction effects (arising from economies of scale, learning-by-doing effects, etc) and product customization to suit different customer categories.

[2] The Coase Conjecture has been analyzed in seminal papers by Bulow (1982) and Stokey (1981). Stokey (1981) studied a monopoly that lacks commitment powers in an infinite-horizon model, and constructed a model with an equilibrium that satisfies the Coase Conjecture; i.e. the monopoly's stock approaches the competitive stock in equilibrium.

[3] Varian (1989) provides an excellent survey of the earlier literature on the subject

[4] This literature includes Bond and Samuelson (1984, 1987), Gul, Sonnenschein and Wilson (1986), Kahn (1986), Suslow (1986), and Ausubel and Deneckere (1989), Besanko and Whinston (1990), Sobel (1991), Olsen (1992), Karp (1996), Kuhn and Padilla (1996) and Kuhn (1998).

(1979), who established that IPD is possible only when certain restrictive conditions regarding the consumers' rate of time preference and their reservation prices hold; Landsberger and Meilijson (1985), who showed that an IPD strategy is optimal for a durable good monopolist if its discount rate is lower than that of consumers; Conlisk, Gerstner and Sobel (1984), who demonstrated that falling prices may result from the entry of new customers as the monopoly chooses to sell to high-valuation consumers first, and then holding periodic sales to clear the market; and Cayseele (1991), who provided a rationing perspective to explain an IPD strategy. More recently, Van Ackere and Reyniers (1995) studied the use of trade-ins and introductory offers to create opportunities for intertemporal price discrimination; Bagnoli, Salant and Swierzcuiski (1995) considered the intertemporal self-selection problem with multiple buyers; and Ault, Randolph, Laband and Saba (2000) studied the optimal usage of rebates as in an IPD sales strategy.[5] In two interesting papers, Rustichin and Villamil (1996, 2000) studied the effects of asymmetric information on the selection of intertemporal pricing strategies.

While numerous studies have identified various institutional factors and market practices that allow a durable-good monopoly to profitably implement IPD sales strategies, there are relatively few studies that try to relate the sources of IPD opportunities to household heterogeneity. Two such studies are Leung (1997), and Rodriguez and Locay (2002).[6] This provides the motivation for the present paper, which investigates the optimality of an IPD sales strategy in a model with the following features: (1) households face an intertemporal budget constraint, which is the present discounted value of lifetime income, (2) households consume both durable goods and a stream of non-durable goods; (3) households differ in the utilities they derive from the consumption of the new or existing stock of durable goods, or their intertemporal budget constraints. The setting we consider is a familiar one in consumer demand theory (see Deaton and Muellbauer, 1980).

Our model is constructed from 'first-principles", i.e. we first derive the household demand for the new durable good beginning with the intertemporal utility maximization problem of a household. We prove that an IPD sales strategy is optimal even when households possess the same rate of discount as the monopoly. Moreover, the optimal price of the durable

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[5] A number of authors such as Sobel (1984), Borenstein (1985), Doyle (1986), Van Praag and Bode (1992) and Locay and Rodriguez (1992) have studied the optimality of IPD in competitive markets. Moorthy and Png (1992) and Dhebar (1994) have analyzed the issue of intertemporal price discrimination and product-quality improvements.

[6] Leung (1997) examined a two-period model in a setting similar to the one we study in this paper. Rodriguez and Locay (2002) also focused on consumer heterogeneity to explain the occurrence of IPD in a two-period model. Firstly, if consumers have different beliefs regarding product availability in the second period, this would lead to differences in the 'effective' discount rates used by consumers. Secondly, if 'snob effects' are present, so that the satisfaction a consumer derives from the product decreases with the number of consumers that have consumed it. Delaying consumption thus reduces the quality of the good, from the consumer's perspective. In both cases, an IPD sales strategy is profitable.

good need not be decreasing over time (as we shall show in an example). Like other forms of sales discrimination, households in our model self-select and reveal their type through their choice of the optimal date of purchase.

The paper is organized as follows. Section 2 describes a model of household consumption subject to an intertemporal budget constraint. In Section 3, we solve for the monopoly's optimal sales strategy and prove that it does not entail a constant price. Next, in Section 4, we show that the optimal price can be increasing over time, as well as provide a sufficient condition for the optimal price to be decreasing over time. Other sources of household heterogeneity are analyzed. Concluding comments are collected in Section 5.

2. A Two-Good Model of Household Consumption

Consider an economy of infinitely-lived households, each consuming at most one unit of a new durable good to be supplied by a monopoly. Households possess an existing stock of durable goods and also consume a stream of non-durable good, subject to an intertemporal budget constraint, denoted by *Y*, which is the present discounted value of lifetime income. We assume that the durable goods are infinitely durable, and the non-durable good is instantaneously perishable once purchased and consumed. The consumption of the two goods is complementary, so that the utility function at time *t*, is

$$(A + s_t X)^{\alpha} q_t^{(1-\alpha)} \qquad (1)$$ [7]

where $s_t = 0$ if the durable good is not purchased at time *t*, and $s_t = 1$ if it has been purchased at *t* or earlier; *X* is the per-period utility that the household derives from the new durable good; q_t is the amount of non-durable good consumed at time *t*. The parameter $A\ (\geq 1)$ is a measure of the utility that households currently derive from their existing durable-good stock, while $\alpha \in (0,\ 1)$ measures the weightage of the consumption of the durable good in the utility function. The supply of the non-durable good is perfectly elastic at a constant unit price.

Let $\{q_{t0}\}$ and $\{q_{t1}\}$ denote, respectively, the household's consumption plan of the non-durable good before and after the purchase of the new durable good. Time is continuous, and we denote *z* as the date of purchase of the durable good, and *r* as the common rate of discount for the households and the monopoly. The present discounted value of a household's utilities is:

$$W(z, \{q_{t0}\}, \{q_{t1}\}, X) = \int_0^z A^{\alpha} q_{t0}^{(1-\alpha)} e^{-rt} dt + \int_z^{\infty} (A + X)^{\alpha} q_{t1}^{(1-\alpha)} e^{-rt} dt \qquad (2)$$

The durable-good monopoly has a constant marginal cost of production, which we shall assume to equal zero, with no loss of generality. At $t = 0$, the monopoly announces a pricing policy $P(t)$, $t \in [0, T]$, and is able to credibly commit to end sales after period *T*.[8]

All households are fully informed about the monopoly's pricing policy. The intertemporal budget constraint of each household is given by

$$P(z)e^{-rz} + \int_0^z e^{-rt} q_{t0} dt + \int_z^\infty e^{-rt} q_{t1} dt \le Y \tag{3}$$

The household's objective is to maximize the net present value of utilities; its maximization problem is given by the following Lagrangian (LP),

$$L(z,\{q_{t0}\},\{q_{t1}\},\lambda) = W(z,\{q_{t0}\},\{q_{t1}\},X) + \lambda\left\{Y - P(z)e^{-rz} - \int_0^z e^{-rt} q_{t0} dt - \int_z^\infty e^{-rt} q_{t1} dt\right\} \tag{4}$$

Differentiating LP with respect to q_{t0} and q_{t1} lead to the following first-order conditions:

$$(1-\alpha)\left[\frac{A}{q_{t0}}\right]^\alpha = (1-\alpha)\left[\frac{A+X}{q_{t1}}\right]^\alpha = \lambda \tag{5}$$

The condition in (5) indicates that q_{t0} is constant for $t=[0,z)$ and q_{t1} is constant for $t=[z,\infty)$. Let $z(X)$ denote the optimal purchase date for the durable good and let $q_0(X)$ and $q_1(X)$ denote, respectively, the optimal per-period consumption of the non-durable good before and after the purchase of the durable good.

Denoting the household's optimal consumption plan by $\{z(X), q_0(X), q_1(X)\}$, the present discounted value of the household's utility under the optimal consumption plan is

$$V(z(X),X) \equiv \frac{1}{r}\left[A^\alpha q_0(X)^{1-\alpha}\left[1-e^{-rz(X)}\right] + \left(A+X\right)^\alpha q_1(X)^{1-\alpha} e^{-rz(X)}\right] \tag{6}$$

and the intertemporal budget constraint is

$$P(z(X))e^{-rz(X)} + \frac{1}{r}\left[q_0(X)\left[1-e^{-rz(X)}\right] + q_1(X)e^{-rz(X)}\right] = Y$$

Utilizing (5), we obtain $q_0(X) = \dfrac{A}{A+X} q_1(X)$ when $z(X) \in (0,T)$. Therefore, the per-period consumption of the non-durable good is higher after the purchase of the durable good. This result is due to consumption complementarities between the durable good and the non-durable good in the non-separable utility function. Next, we obtain

$$V(z(X),X) = \left[\frac{A+Xe^{-rz(X)}}{r}\right]^\alpha \left[Y - P(z(X))e^{-rz(X)}\right]^{(1-\alpha)} \tag{7}$$

$$q_1(X) = \frac{r\left(A+X\right)}{A+Xe^{-rz(X)}}\left[Y - P(z(X))e^{-rz(X)}\right] \tag{8}$$

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[7] While the analysis considers a non-separable utility function, it can be easily generalized to other classes of utility functions.

[8] In practice, the monopoly's credibility in committing to a pricing policy depends on factors such as the monopoly's reputation, industry practice and whether the monopoly will undertake actions to make it costly to renege on its commitment. For instance, if the monopoly continuously launches new products, the credibility of its future sales strategies will be affected by its reputation.

$$\lambda = (1-\alpha)\left[\frac{A + Xe^{-rz(X)}}{r\left[Y - P(z(X))e^{-rz(X)}\right]}\right]^{\alpha} \tag{9}$$

Here, λ is the marginal utility from consuming the non-durable good, after the optimal purchase date of the durable good, $z(X)$, has been determined.

3. The Monopoly's Optimal Sales Strategy

The model considered in this paper applies to durable goods that cannot be produced by competing firms now or in the future. Some examples that closely fit this model include electronic equipment that is not compatible with that produced by other firms and which a firm might reasonably find it profitable to eventually discontinue; for instance, some types of computer games, Iomega JAZZ and ZIP drives would fall into this category. Another example that closely fits our model is the pricing of new Volkswagen Beetle or the new Mini Cooper, as well as any other limited edition niche automobile whose concept cannot be effectively replicated by other firms for historical reasons.

We shall consider the case where households are heterogeneous in the per-period utility derived from the consumption of the new durable good (*X*), but are otherwise identical. The density function of the household types is *g*(*X*) over the support $[X^-, X^+]$; this determines the level of demand for the durable good at each value *X*.[9]

For an IPD sales strategy to be feasible, the monopoly must be able implement a pricing policy *P*(*t*) under which households with a higher *X* would purchase it earlier. We shall restrict our analysis to pricing policies such that in no time subinterval between 0 and *T*, sales are zero. By differentiating *LP* in (4) with respect to *z*, we obtain the first-order condition for *z*(*X*) when it is an interior solution; i.e.

$$\beta r X\left[\frac{Y - P(z(X))e^{-rz(X)}}{A + Xe^{-rz(X)}}\right] = rP(z(X)) - \left.\frac{dP(t)}{dt}\right|_{t=z(X)} \tag{10}$$

where $\beta \equiv \alpha/(1-\alpha)$ henceforth in our analysis. The optimal time of purchase, *z*(*X*), is chosen such that the present discounted value of the marginal gain in utility from the consumption of the non-durable good at *z*(*X*) is equal to the marginal loss in utility from delaying the consumption of the new durable good. Implicitly differentiating *z*(*X*) with respect to *X* in (10), and utilizing the conditions in (10) and (11), it is straightforward to verify that $dz(X)/dX < 0$. Thus, under a feasible IPD strategy implemented by the monopoly, households who derive

9 In Section 4.3, we consider other sources of household heterogeneity.

higher per-period utility from the durable good will purchase it earlier. To ensure $z(X)$ is unique, the necessary second-order condition for $z(X)$ can be shown to be the following:

$$r\frac{dP(t)}{dt}\bigg|_{t=z(X)} - \frac{d^2P(t)}{dt^2}\bigg|_{t=z(X)} < 0 \tag{11}$$

Let $X_L \equiv \text{Min}\{X \geq X^- \mid z(X)$ is an interior solution$\}$ and $X_H \equiv \text{Max}\{X \leq X^+ \mid z(X)$ is an interior solution$\}$. Since $z(X)$ is decreasing in X, we have $z(X_L) = T$ and $z(X_H) = 0$, if X_L and X_H exist. Define $X_M \in [X^-, X_L]$ as the minimum X such that a household with $X = X_M$ is indifferent between buying the new durable good at time T, given $P(T)$, or not buying it at all, and households with $X < X_M$ would rather do without the new durable good. Given the pricing policy $P(t)$, the timing of the purchase of the durable good is as follows:

$$\begin{array}{lll} X \in [X_H, X^+], & z(X) = 0 & \\ X \in (X_L, X_H), & z(X) \in (0,T), \quad dz(X)/dX < 0 & \\ X \in [X_M, X_L], & z(X) = T & \\ X \in [X^-, X_M), & z(X) = \infty \quad \text{(i.e. the durable good is not purchased at all)} & \end{array} \tag{12}$$

Since $z(X)$ is monotonic in X, we can invert $z(X)$ to define a purchase schedule $X(t)$, implicitly chosen by the monopoly for a particular pricing policy $P(t)$, such that

$$\begin{array}{ll} t = 0, & X(t) \in [X_H, X^+] \\ t \in (0,T), & X(t) = z^{-1}(t) \in (X_L, X_H) \\ t = T, & X(t) \in [X_M, X_L] \\ t = \infty, & X(t) \in [M^-, X_M) \end{array} \tag{13}$$

where $X(t)$ satisfies, when it is an interior solution,[10]

$$\beta r X(t)\left[\frac{Y - P(t)e^{-rt}}{A + X(t)e^{-rt}}\right] = rP(t) - \frac{dP(t)}{dt} \tag{14}$$

$$\frac{dX(t)}{dt} = \frac{\frac{1}{\beta}\left[r\frac{dP(t)}{dt} - \frac{d^2P(t)}{dt^2}\right] - rX(t)e^{-rt}\left[rP(t) - \frac{dP(t)}{dt}\right]\left[A + Xe^{-rt}\right]^{-1}}{Ar\left[Y - P(t)e^{-rt}\right]\left[A + Xe^{-rt}\right]^{-2}} < 0 \tag{15}$$

Proposition 1: A feasible IPD pricing policy $P(t)$, $t \in [0,T]$, is a function of the monopoly's choice of purchase schedule $X(t)$, the customer base (through the choice of the marginal household type X_M), and the sales period T. It is given by

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[10] Note that $X(t)$ is multi-valued at $t = 0$ (with X_H as an interior solution) and at $t = T$ (with X_L as an interior solution). Without loss of generality to the analysis, we further assume that $X(t)$ is continuous and is continuously differentiable.

$$P(t) = e^{rt} Y \left\{ 1 - \left[\frac{A}{A + X_M e^{-rT}} \right]^{\beta} \exp\left\{ -\beta \int_t^T \frac{rX(z)e^{-rz}}{A + X(z)e^{-rz}} dz \right\} \right\} \tag{16}$$

Proof: A household who values the new durable good at X_M is indifferent between purchasing it or going without it, so that its utility is given by $V(T, X_M) = \left[\frac{A}{r}\right]^{\alpha} Y^{(1-\alpha)}$. Using (7), we establish *P*(*T*) as a boundary condition of the pricing policy *P*(*t*):

$$P(T) = e^{rT} Y \left\{ 1 - \left[\frac{A}{A + X_M e^{-rT}} \right]^{\beta} \right\} \tag{17}$$

Next, *P*(*t*) is derived using the differential equation in (14) and the boundary condition of *P*(*T*) in (17). The detailed derivation of *P*(*t*) is provided in the Appendix. *Q.E.D.*

A couple of remarks are in order. Firstly, it is straightforward to verify that the feasible IPD price policy, *P*(*t*), includes the constant price as a feasible strategy. Secondly, note that while $e^{-rt}P(t)$ must be decreasing in *t*, as is evident from (16), it is possible for $P(t)$ to be increasing over time, and for households to optimally decide to purchase the durable good later and pay a higher price. This result contrasts with the findings in Stokey (1979) and Landsberger and Meilijson (1985), where only a durable good is consumed and consumers are not subject to an intertemporal budget constraint. Specifically, Equation (6) of Landsberger and Meilijson (1985) requires that the pricing policy satisfy $dP(t)/dt = r\left[P(t) - X(t)\right]$. Since $P(t)$ must necessarily be less than $X(t)$, the utility derived by the consumer from the durable good, it follows that $dP(t)/dt < 0$ in the model of Landsberger and Meilijson (1985).

The intuition that, in our model, it may be optimal for households to purchase the new durable good at a later date, even though the price is non-decreasing over time, is as follows. Suppose a household is deciding between whether to purchase the durable good at time *t* or $t+\Delta t$, and the price $P(t)$ is non-decreasing over the time interval $[t,\ t+\Delta t]$. If the household decides to purchase the durable good at time *t*, the available budget for purchasing the stream of non-durable good is $\left[Y - P(t)e^{-rt}\right]$, while if the decision is to postpone the purchase to time $t+\Delta t$ instead, the available budget is now $\left[Y - P(t+\Delta t)e^{-r(t+\Delta t)}\right]$. By Equation (16), we know that $e^{-rt}P(t)$ must be strictly decreasing in *t*. Hence, it follows that purchasing the durable good at time $t+\Delta t$ leaves the household with a larger budget for consuming the non-durable good, compared with a decision to purchase the durable good at *t*. Thus, the household is able to consume a larger stream of non-durable good, both before and after the purchase of the durable good if the purchase is made at time $t+\Delta t$. Since per period utility of the household is the product of (a concave function of) the consumption of the non-durable good and the services obtained from the durable good, a decision to purchase the durable good later can be preferable, if discounted marginal gain in utility of consuming a larger stream of the non-durable good outweighs the marginal loss from delaying the consumption of the new durable good.

We turn now to the derivation of the optimal pricing strategy of the firm. We shall prove shortly that the optimal price path may be increasing or decreasing over time, and that charging a constant price is not optimal. Before we proceed to the formal derivation of the optimal pricing strategy, we first need to derive the present discounted value of the monopoly's revenue as a function of its choice of the purchase schedule $X(t)$, the marginal household (X_M) and the sales period T. The monopoly's revenue function can be shown to be given by

$$\Pi(X(t), X_M, T) \equiv Y\beta\left[\frac{A}{A+X_M e^{-rT}}\right]^{\beta}\int_0^T\left\{\frac{re^{-rt}X(t)\left[1-G(X(t))\right]}{A+X(t)e^{-rt}}\exp\left\{-\beta\int_t^T\frac{rX(z)e^{-rz}}{A+X(z)e^{-rz}}dz\right\}\right\}dt$$
$$+Y\left[1-G(X_M)\right]\left\{1-\left[\frac{A}{A+X_M e^{-rT}}\right]^{\beta}\right\} \tag{18}$$

(The derivation is provided in the Appendix.) Let $\{X^*(t), X_M^*, T^*\}$ denote the monopoly's optimal sales strategy. We assume $[X^-, X^+]$ is sufficiently wide so that X_M^* is an interior solution. Differentiating $\Pi(X(t), X_M, T)$ with respect to $X(t)$, X_M and T, we obtain the first-order conditions to characterize the optimal sales strategy and the monopoly's maximized revenue. The first-order conditions are given in (A5), (A6) and (A7) in the Appendix. We obtain the following Propositions:

Proposition 2: The present value of the monopoly's revenue under the optimal sales strategy is

$$\Pi(X^*(t), X_M^*, T^*) = \frac{\left\{\left[A+X_M^* e^{-rT^*}\right]\left\{1-\left[\frac{A}{A+X_M^* e^{-rT^*}}\right]^{\beta}\right\}+\beta\left[1-e^{-rT^*}\right]X_M^*\right\}\left[1-G(X_M^*)\right]Y}{A+X_M^* e^{-rT^*}+\beta\left[1-e^{-rT^*}\right]X_M^*} \tag{19}$$

where $\{X^*(t), X_M^*, T^*\}$ satisfies the following set of conditions, with $X_L \equiv X^*(T^*)$:

$$\frac{X_L\left[1-G(X_L)\right]}{A+X_L e^{-rT*}} = \frac{X_M^* e^{rT*}}{\beta}\left\{\left[\frac{A+X_M^* e^{-rT^*}}{A}\right]^{\beta}-1\right\}g(X_M^*) \tag{20}$$

$$\frac{1-G(X_M^*)}{g(X_M^*)} = \frac{e^{rT*}}{\beta}\left\{\left[\frac{A+X_M^* e^{-rT^*}}{A}\right]^{\beta}-1\right\}\left\{A+X_M^* e^{-rT^*}+\beta\left[1-e^{-rT^*}\right]X_M^*\right\} \tag{21}$$

Proof: See Appendix.

Proposition 3: The optimal pricing strategy for the monopoly entails intertemporal price discrimination when the optimal sales strategy $\{X^*(t), X_M^*, T^*\}$ is implemented.

Proof: A simple way to prove Proposition 3 is to construct a contradiction. Since the set of feasible IPD pricing policy $P(t)$ includes a constant price as an option (Proposition 1), we only need to show that a constant price is not part of the optimal sales strategy. Suppose, to the contrary, the optimal sales strategy $\{X^*(t), X_M^*, T^*\}$ entails a constant price, denoted P^*, over the sales period T^*. By the definition of X_M^*, we establish, from (15), that

$$P^* = e^{rT^*} Y\left\{1 - \left[\frac{A}{A + X_M^* e^{-rT^*}}\right]^\beta\right\} \tag{22}$$

so that, utilizing (14) and noting $dP(t)/dt = 0$, $X^*(t)$ is given by:

$$X^*(t) = Ae^{rT^*}\left\{1 - \left[\frac{A}{A + X_M^* e^{-rT^*}}\right]^\beta\right\}\left\{\beta - (\beta+1)e^{r(T^*-t)}\left\{1 - \left[\frac{A}{A + X_M^* e^{-rT^*}}\right]^\beta\right\}\right\}^{-1} \qquad (23)^{11}$$

It follows from (23) that:

$$X_L = X^*(T^*) = Ae^{rT^*}\left\{1 - \left[\frac{A}{A + X_M^* e^{-rT^*}}\right]^\beta\right\}\left\{(\beta+1)\left[\frac{A}{A + X_M^* e^{-rT^*}}\right]^\beta - 1\right\}^{-1} \tag{24}$$

Next, substituting $X^*(t)$ derived in (23) into the revenue function in (18), we obtain (see the Appendix for the derivation):

$$\Pi(X^*(t), X_M^*, T^*) = Y\left\{1 - \left[\frac{A}{A + X_M^* e^{-rT^*}}\right]^\beta\right\}\left\{1 - G(X_M^*) + \int_0^{T^*} re^{r(T^*-t)}\left[1 - G(X^*(t))\right]dt\right\} \tag{25}$$

If the optimal sales strategy entails a constant price, then the revenue function derived in (25) must be equivalent to the one derived in (19). Utilizing (20) and (21) and the equivalent expression of X_L given in (24), straightforward manipulation of (19) yields:[12]

$$\Pi(X^*(t), X_M^*, T^*) = Y\left\{1 - \left[\frac{A}{A + X_M^* e^{-rT^*}}\right]^\beta\right\}\left\{1 - G(X_M^*) + \left[e^{rT^*} - 1\right]\left[1 - G(X_L)\right]\right\} \tag{26}$$

Comparing (25) and (26), this leads to the contradiction that

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[11] Substituting the purchase schedule $X^*(t)$ derived in (23) into the pricing policy $P(t)$ in (16), it is straightforward to verify that the constant-price policy is indeed a feasible strategy for the monopoly.

[12] From the derivation of (19) earlier, we obtain (A10) as an interim step. Substituting (24) into (A10), this gives us the expression of $\Pi(X^*(t), X_M^*, T^*)$ in (26).

$$\left[1-G(X_L)\right]\left[1-e^{-rT^*}\right]=\int_0^{T^*} re^{-rt}\left[1-G(X^*(t))\right]dt \tag{27}$$

To see this, we note from (23) that $X_L = X^*(T^*) \le X^*(t)\ \ \forall\, t < T^*$, so that $\left[1-G(X_L)\right] \ge \left[1-G(X^*(t))\right]$. Thus the quantity on the left-hand side of (27) is larger than the quantity on the right-hand side of (27). This contradiction implies the optimal sales strategy does not involve a constant-price policy, as we had assumed at the outset of the proof. This completes the proof that the optimal sales strategy involves intertemporal price discrimination. *Q.E.D.*

It is instructive to relate this result to the models of Stokey (1979) and Landsberger and Meilijson (1985), where consumers are not constrained by an intertemporal budget. In their models, as long as the consumers and the monopoly share the same rate of discount, an IPD sales strategy is dominated by a constant-price policy. In our model, households face an intertemporal budget constraint, so that an IPD strategy is optimal for the monopoly even though it possesses the same rate of discount as the households. Like other forms of sales discrimination, households in our model self-select and reveal their type (the per-period utility they derive from the consumption of the durable good) through their optimal date of purchase.

4. Variants of the Base Model

4.1 An example of Increasing Prices

In our model, it is possible for the optimal price of the durable good to be decreasing or increasing over time. Here, we provide an example to show the optimal price is increasing over time. Let $X^- = 0$ and $X^+ = 1$, and suppose $g(\ .\ )$ is uniform over [0, 1]. Furthermore, let $A = 1$, and $\alpha = 0.5$ (so that $\beta = 1$). Applying these parameters to the optimality condition in (21), we obtain $X_M^* = \sqrt{2} - 1 < 0.5$. Hence, the monopoly sells to the household segment $[\sqrt{2} - 1, 1]$. Next, substituting the first-order condition for $X^*(t)$ in (A5) into the expression for $P(t)$ in (16), we derive the optimal pricing policy, denoted $P^*(t)$:

$$P^*(t)=e^{rt}Y\left\{1-\frac{\left[X_M^*\right]^2}{1+X_M^*e^{-rT^*}}\frac{1+X^*(t)e^{-rt}}{X^*(t)\left[1+X^*(t)\right]}\right\} \tag{28}$$

where we have made use of the fact that the optimality condition in (20) implies $X_L\left[1-X_L\right]=\left[X_M^*\right]^2\left(1+X_Le^{-rT*}\right)$ under the current specification. Next, differentiating $P^*(t)$ with respect to t, we obtain

$$\frac{dP^*(t)}{dt}=rP^*(t)-\frac{e^{rt}Y\left[X_M^*\right]^2}{1+X_M^*e^{-rT^*}}\left[\frac{e^{-rt}\left[-rX^*(t)+D(t)\right]}{X^*(t)\left[1+X^*(t)\right]}-\frac{\left[1-2X^*(t)\right]\left[1+X^*(t)e^{-rt}\right]D(t)}{\left\{X^*(t)\left[1+X^*(t)\right]\right\}^2}\right] \tag{29}$$

where $D(t) \equiv dX^*(t)/dt < 0$, by (15). Since the monopoly sells to the household segment [0.414, 1], it is easy to see that for $X^*(t) > 0.5$, $dP^*(t)/dt > 0$ in (29). Hence, the optimal price is increasing over time as $X^*(t)$ falls from 1 to 0.5. Next, from the second-order condition in (11), it is clear that if $dP^*(t)/dt > 0$, it must be the case that $d^2P^*(t)/dt^2 > 0$ as well. Hence, we can conclude that $dP^*(t)/dt > 0$, for all $t \in [0, T^*]$.

4.2 Sufficient Condition for Optimal Price to be Decreasing Over Time

To complete the analysis, let us consider a sufficient condition under which $dP^*(t)/dt <$ 0 for all $t \in [0, T^*]$. First, we note from the first-order condition in (14) that

$$\frac{dP^*(t)}{dt} = rP^*(t) - \beta r X^*(t)\left[\frac{Y - P^*(t)e^{-rt}}{A + X^*(t)e^{-rt}}\right]$$

Hence, if $dP^*(t)/dt < 0$, this implies the following condition must hold:

$$P^*(t) < \frac{\beta X^*(t)Y}{A + (\beta+1)X^*(t)e^{-rt}} \equiv W(t) \tag{30}$$

Next, we note that $e^{rt}Y - W(t) > 0$. As $\alpha \to 1$, so that $\beta \to \infty$, $e^{rt}Y - W(t)$ tends to 0. Hence, the condition in (30) converges to the budget constraint that $P^*(t)e^{-rt} < Y$. Therefore, if α, the weightage place on the consumption of the durable good in the utility function, is sufficiently close to 1, the optimal price of the durable good will be decreasing over time.

4.3 Other Sources of Household Heterogeneity

Besides heterogeneity in the utility derived from the consumption of the durable good (X), two other sources of household heterogeneity that could give rise to an optimal IPD strategy are the utility they derive from the existing stock of durable goods (A) and the intertemporal constraint (Y). Household heterogeneity in either A or Y can be shown to yield qualitatively the same results as in the case of heterogeneity in X.

First, a given distribution of A can be shown to map into an equivalent distribution of X. To see this, suppose that purchasing the new durable good increases the first term of the household's utility function in (1) by an amount $\Delta V \equiv (A + X)^\alpha - A^\alpha$ There exists a value of X, given by $X = (A^\alpha + \Delta V)^{\alpha^{-1}} - A$, such that any change in the parameter A can be expressed as an equivalent change in parameter X, with the same impact on the household's utility. Therefore, a distribution of X can be mapped into an equivalent distribution of A, and the results regarding the optimality of an IPD sales strategy carry over to the case of heterogeneity in A.

An IPD sales strategy is also optimal in the case of heterogeneity in the intertemporal budget constraint (*Y*). Similarly, a given distribution in *Y* can be shown to map into an equivalent distribution in *X*. The equivalence can be observed with reference to Equation (7), which gives the optimized discounted utility of an infinitely-lived household. Variations in the values of X and the accompanying optimal date of purchase $z(X)$ result in changes to $V(z(X), X)$. Suppose we consider a change in X to $X + \Delta X$, and let $z(X + \Delta X) \equiv z^*$ and $V(z(X + \Delta X), X + \Delta X) \equiv V^*$. Using Equation (7), it is easy to verify that we can achieve the same magnitude of increase in utility to V^* by keeping X fixed and increasing Y appropriately, so that the optimal timing of purchase is at z^*. It follows that for a given mapping of *Y*, there exists an equivalent mapping of *X* that can give rise to the same optimal time of purchase $z(X)$ and the optimized utility $V(z(X), X)$.

5. Conclusion

This paper studies the optimality of intertemporal price discrimination for a durable-good monopoly in a model where households consume both durable goods and non-durable goods subject to an intertemporal budget constraint. The choice of the optimal date of purchase of the durable good affects the budget share for the consumption of the stream of non-durable good, as well as the marginal utility of consumption. Households differ in the utility they derive from the consumption of the durable good; those with lower valuations optimally choose to purchase the durable good at a later date as the discounted marginal gain in utility from consuming a larger stream of the non-durable good outweighs the marginal loss from delaying the consumption of the durable good. This results in profitable intertemporal price discrimination for the monopoly. As in the case of static price discrimination, households self-select and reveal their type through their choice of the date of purchase of the durable good.

In our model, the optimal pricing strategy does not always entail a decreasing price. It is possible for the optimal price to increase over time, as we demonstrate in the case of a uniform distribution of household types and equal weightage placed on the consumption of the durable good and the non-durable good in the household's utility function. We also show that when the weightage placed on the durable good is sufficiently close to one, the optimal price is decreasing over time.

An interesting issue that is not considered in this paper is the optimal intertemporal pricing strategy in the presence of temporary patent protection. This environment differs from the one analyzed in this paper in that the terminal time of the monopoly, *T*, is exogenously determined by the patent's expiry date. Moreover, if the absence of patent protection leads to a competitive market, consumers would be able to purchase the durable good at marginal cost after the patent protection expires. In this paper, we assume that consumers do not have the option to purchase after time *T*. This extension is left for future research.

Appendix

Derivation of P(t) in (16): We can rewrite the first-order condition in (14) as follows :

$$\left[\frac{rP(t)-\dfrac{dP(t)}{dt}}{Y-P(t)e^{-rt}}\right]=\frac{\beta rX(t)}{A+X(t)e^{-rt}} \tag{A1}$$

Integrating both sides of (A1) with respect to *t*,

$$\ln\left[Y-P(t)e^{-rt}\right]=-\beta\int_t^T\frac{rX(z)e^{-rz}}{A+X(z)e^{-rz}}dz+K$$

where *K* is a constant term following the integration. Noting the boundary condition $P(T)$ as given in (17), we obtain $e^K=Y\left[\dfrac{A}{A+X_Me^{-rT}}\right]^{\beta}$, which leads to $P(t)$ in (16).

Derivation of the monopoly's revenue function in (18):

We begin by noting that the present value of the monopoly's revenue is defined as

$$\int_{M^-}^{M^+}e^{-rz(X)}P(z(X))g(X)dX=\int_{X_H}^{M^+}P(0)g(X)dX+\int_{X_M}^{X_L}e^{-rT}P(T)g(X)dX+\int_{X_L}^{X_H}e^{-rz(X)}P(z(X))g(X)dX$$

Integrating by parts and using the fact that – $g(X)$ is the derivative of [1 – G(X)], we can rewrite the revenue function as:

$$\left[1-G(X_M)\right]e^{-rT}P(T)+\int_{X_L}^{X_H}e^{-rz(X)}\left[rP(z(X))-\frac{dP(t)}{dt}\bigg|_{t=z(X)}\right]\frac{dz(X)}{dX}\left[1-G(X)\right]dX \tag{A2}$$

Using the first-order condition for $z(X)$ in (7), and a change in variable, writing $z(X)=t$, so that $dz(X)/dX=dt$, further manipulation of (A2) yields

$$\left[1-G(X_M)\right]e^{-rT}P(T)+\beta\int_0^T\frac{re^{-rt}X(t)\left[1-G(X(t))\right]}{A+X(t)e^{-rt}}\left[Y-P(t)e^{-rt}\right]dt \tag{A3}$$

Upon substituting $P(T)$ and $P(t)$ into (A3), we derive the present value of the monopoly's revenue as presented in (18).

Derivation of first-order conditions for $\{X^*(t),X_M^*,T^*\}$:

To characterize $X^*(t)$, we differentiate $\Pi(X(t),X_M,T)$ with respect to $X(t)$ to obtain the following first-order condition:

$$\frac{\left[A+X^*(t)e^{-rt}\right]g(X^*(t))X^*(t)-A\left[1-G(X^*(t))\right]}{\left[A+X^*(t)e^{-rt}\right]^2}=\frac{\beta Are^{-rt}}{\left[A+X^*(t)e^{-rt}\right]^2}\left[\frac{X^*(t)\left[1-G(X^*(t))\right]}{A+X^*(t)e^{-rt}}\right]$$

which can be re-written as:

$$\frac{d}{dX^*(t)}\left\{\ln\left[\frac{X^*(t)\left[1-G(X^*(t))\right]}{A+X^*(t)e^{-rt}}\right]\right\} = \frac{\beta Are^{-rt}}{\left[A+X^*(t)e^{-rt}\right]^2} \tag{A4}$$

Integrating both sides of (A4) with respect to $X^*(t)$,

$$\ln\left[\frac{X^*(t)\left[1-G(X^*(t))\right]}{A+X^*(t)e^{-rt}}\right] = \beta\int_t^T \frac{rX^*(z)e^{-rz}}{A+X^*(z)e^{-rz}}dz + K$$

where K here refers to the constant term following the integration. Noting the boundary condition $X^*(T) = X_L$, we have $K = \frac{X_L\left[1-G(X_L)\right]}{A+X_L e^{-rT}}$, which leads to

$$\frac{X^*(t)\left[1-G(X^*(t))\right]}{A+X^*(t)e^{-rt}} = \frac{X_L\left[1-G(X_L)\right]}{A+X_L e^{-rT}}\exp\left\{\beta\int_t^T \frac{rX^*(z)e^{-rz}}{A+X^*(z)e^{-rz}}dz\right\} \quad t\in[0,T] \tag{A5}$$

Differentiating $\Pi(X(t), X_M, T)$ with respect to X_M yield the first-order condition for X_M^*:

$$\beta\int_0^T\left\{\frac{re^{-rt}X(t)\left[1-G(X(t))\right]}{A+X(t)e^{-rt}}\exp\left\{-\beta\int_t^T\frac{rX(z)e^{-rz}}{A+X(z)e^{-rz}}dz\right\}\right\}dt = \tag{A6}$$

$$1-G(X_M^*)-\frac{\left[A+X_M^*e^{-rT}\right]^{\beta+1}}{\beta A^\beta e^{-rT}}\left\{1-\left[\frac{A}{A+X_M^*e^{-rT}}\right]^\beta\right\}g(X_M^*)$$

Lastly, we obtain the following first-order condition for T^*: (A7)

$$\beta\int_0^{T^*}\left\{\frac{re^{-rt}X(t)\left[1-G(X(t))\right]}{A+X(t)e^{-rt}}\exp\left\{-\beta\int_t^{T^*}\frac{rX(z)e^{-rz}}{A+X(z)e^{-rz}}dz\right\}\right\}dt = 1-G(X_M)-\left[\frac{A+X_M e^{-rT^*}}{A+X_L e^{-rT^*}}\right]\frac{X_L}{X_M}\left[1-G(X_L)\right]$$

These first-order conditions allow us to characterize the monopoly's revenue and the optimal sales strategy in Proposition 2 as follows. First, substituting (A5) into (A6), we get

$$\beta\frac{X_L\left[1-G(X_L)\right]}{A+X_L e^{-rT^*}}\left[1-e^{-rT^*}\right] = 1-G(X_M^*)-\frac{A+X_M^*e^{-rT^*}}{\beta e^{-rT^*}}\left\{\left[\frac{A+X_M^*e^{-rT^*}}{A}\right]^\beta-1\right\}g(X_M^*) \tag{A8}$$

Substituting (A5) into (A7), we get.

$$\beta\frac{X_L\left[1-G(X_L)\right]}{A+X_L e^{-rT^*}}\left[1-e^{-rT^*}\right] = 1-G(X_M^*)-\left[\frac{A+X_M^*e^{-rT^*}}{A+X_L e^{-rT^*}}\right]\frac{X_L}{X_M^*}\left[1-G(X_L)\right] \tag{A9}$$

Using (A8) and (A9), we derive (20). Next, using (20), and (A9), we derive (21).

Derivation of the revenue function in (19): Using (A5), we can write (18) as follows

(A10)

$$\Pi(X^*(t), X_M^*, T^*) \equiv \beta Y\left[\frac{A}{A+X_M^*e^{-rT^*}}\right]^\beta\left\{\frac{X_L\left[1-G(X_L)\right]}{A+X_L e^{-rT^*}}\right\}\int_0^{T^*}re^{-rt}dt + Y\left[1-G(X_M^*)\right]\left\{1-\left[\frac{A}{A+X_M^*e^{-rT^*}}\right]^\beta\right\}$$

Next using (20), the first term simplifies further to

$$\beta Y\left[\frac{A}{A+X_M^* e^{-rT^*}}\right]^{\beta}\left\{\frac{X_M^* e^{rT*}}{\beta}\left\{\left[\frac{A+X_M^* e^{-rT^*}}{A}\right]^{\beta}-1\right\}g(X_M^*)\right\}\left[1-e^{-rT^*}\right]$$

Utilizing (21), we can simplify the above expression

$$\beta Y\left[\frac{A}{A+X_M^* e^{-rT^*}}\right]^{\beta}\frac{X_M^*\left[1-G(X_M^*)\right]\left[1-e^{-rT^*}\right]}{A+X_M^* e^{-rT^*}+\left[1-e^{-rT^*}\right]\beta X_M^*}$$

Therefore, rearranging, we obtain an expression for $\Pi(X^*(t),X_M^*,T^*)$ as stated in (19).

Derivation of the expression in (25): Using (23), we obtain:

$$\frac{\beta r X^*(z)e^{-rz}}{A+X^*(z)e^{-rz}}=\frac{\beta r A e^{r(T^*-z)}\left\{1-\left[\frac{A}{A+X_M^* e^{-rT^*}}\right]^{\beta}\right\}}{A\left\{\beta-(\beta+1)e^{r(T^*-z)}\left\{1-\left[\frac{A}{A+X_M^* e^{-rT^*}}\right]^{\beta}\right\}\right\}+Ae^{r(T^*-z)}\left\{1-\left[\frac{A}{A+X_M^* e^{-rT^*}}\right]^{\beta}\right\}}$$

$$=\frac{re^{r(T^*-z)}\left\{1-\left[\frac{A}{A+X_M^* e^{-rT^*}}\right]^{\beta}\right\}}{1-e^{r(T^*-z)}\left\{1-\left[\frac{A}{A+X_M^* e^{-rT^*}}\right]^{\beta}\right\}} \qquad \text{(A11)}$$

Hence, $-\beta\int_t^{T^*}\frac{rX^*(z)e^{-rz}}{A+X^*(z)e^{-rz}}dz=-\left[\ln\left\{1-e^{r(T^*-z)}\left\{1-\left[\frac{A}{A+X_M^* e^{-rT^*}}\right]^{\beta}\right\}\right\}\right]_t^{T^*}$

so that $$\exp\left\{-\beta\int_t^{T^*}\frac{rX^*(z)e^{-rz}}{A+X^*(z)e^{-rz}}dz\right\}=\frac{1-e^{r(T^*-t)}\left\{1-\left[\frac{A}{A+X_M^* e^{-rT^*}}\right]^{\beta}\right\}}{\left[\frac{A}{A+X_M^* e^{-rT^*}}\right]^{\beta}} \qquad \text{(A12)}$$

Therefore, using (A1) and (A12), we can write the first term of (18) as follows:

$$= Y\beta\left[\frac{A}{A+X_M^* e^{-rT^*}}\right]^{\beta}\int_0^{T^*}\left\{re^{-rt}\left[1-G(X^*(t))\right]\left\{\frac{e^{rT^*}\left\{1-\left[\frac{A}{A+X_M^* e^{-rT^*}}\right]^{\beta}\right\}}{\left[\frac{A}{A+X_M^* e^{-rT^*}}\right]^{\beta}}\right\}\right\}dt \qquad \text{(A13)}$$

This then allows the derivation of the expression stated in (25).

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